



CIEE Global Institute - Berlin

Course name:	Calculus II
Course number:	(GI) MATH 1002 BRGE
Programs offering course:	Berlin Open Campus Block
Open Campus track:	STEM and Society
Language of instruction:	English
U.S. semester credits:	3
Contact hours:	45
Term:	Spring 2020

Course Description

Students will learn the basic concepts of Calculus typical for the second semester of a two or three semester sequence: Integration, Differential Equations, Sequences and Series, Power Series, Parametric Equations and Polar Coordinates. Applications are drawn from many areas of science and engineering: biomechanics, ecology, epidemiology, genetics, medicine, pharmacology, physiology, and others. For example, in conservation biology, calculus measures the effect of habitat fragmentation on population dynamics; in epidemiology, calculus tracks antigenic changes in an influenza epidemic; in physiology, calculus shows how blood pressure depends on the radius of an artery.

Learning Objectives

Upon completion, students taking this course will be able to:

- Evaluate definite and indefinite integrals using techniques including change of variables, integration by parts and the Integral Table.
- Apply integration to find volumes of solids, consumer surplus, producer surplus and exponential growth and decay.
- Evaluate double integrals of functions of several variables.
- Solve simple and separable differential equations.
- Describe curves parametrically and via polar equations.
- Construct graphs of common functions using Calculus.
- Employ convergence and divergence tests to analyze behavior of infinite series and sequences.
- Represent functions using power series and solve problems using them.
- Describe lines and planes in space in equation form and solve problems using them.



- Develop models to apply Calculus to everyday phenomena.
- Articulate intuitively how Calculus works and its importance to local and global society.

Course Prerequisites

Calculus 1

Methods of Instruction

Students will attend lectures, problem-solving workshops, discussions and excursions. Lectures will emphasize theory and applications. Considerable time will be spent solving problems individually and in groups with instructor oversight. In addition, students will be given narratives to translate mathematically and compute their solutions. They will discuss the relevance of Calculus to a host of real-life situations involving physics, engineering, architecture, conservation, public health and human physiology. Excursions will explore the use of Calculus in real life, including the local culture.

Assessment and Grading

Participation	20%
Weekly Quizzes (5)	30%
Problem Sets	30%
Lecture Activities	10%
Excursion Essays (2)	10%
Total	100%

Course Requirements

Weekly Quizzes

Each week, students will take a quiz on the previous week's course material, including lectures, activities and readings. Quizzes will have True/False, Multiple Choice, calculations, filling in blanks and short answer questions. Quizzes will cover only new material from that week but will build on previous concepts.

Problem Sets

Each session will include a set of problems for students to solve individually and in groups. Certain of these will be handed in a graded. Grading will include both the ultimate solution, the student's reasoning in solving the problem and the student's ability to clearly and intuitively explain the problem and its solution.

Lecture Activities



After each lecture, students will have a series of tasks and demonstrations related to the lecture material. They will work in groups to complete the tasks, handing in answers to a series of questions before leaving the class.

Excursion Essays

Some sessions will leave the classroom and enter Mérida's historic center and areas around the city. On excursions, students will visit sites where Calculus is used every day. They will interact with local mathematicians, construction workers, business owners and others to explore Calculus in the local culture. After these visits, students will select one application and write a 500 word essay to communicate how Calculus is used in that context to solve a real life problem.

Participation

Participation is valued as meaningful contribution in the digital and tangible classroom, utilizing the resources and materials presented to students as part of the course. Meaningful contribution requires students to be prepared in advance of each class session and to have regular attendance. Students must clearly demonstrate they have engaged with the materials as directed, for example, through classroom discussions, online discussion boards, peer-to-peer feedback (after presentations), interaction with guest speakers, and attentiveness on co-curricular and outside-of-classroom activities.

Class Attendance

Regular class attendance is required throughout the program, and all unexcused absences will result in a lower participation grade for any affected CIEE course. Due to the intensive schedules for Open Campus programs, unexcused absences that constitute more than 10% of the total course will result in a written warning.

Students who transfer from one CIEE class to another during the add/drop period will not be considered absent from the first session(s) of their new class, provided they were marked present for the first session(s) of their original class. Otherwise, the absence(s) from the original class carry over to the new class and count against the grade in that class.

For CIEE classes, excessively tardy (over 15 minutes late) students must be marked absent. Attendance policies also apply to any required co-curricular class excursion or event, as well as to Internship, Service Learning, or required field placement. Students who miss class for personal travel, including unforeseen delays that arise as a result of personal travel, will be marked as absent and unexcused. No make-up or re-sit opportunity will be provided.

Attendance policies also apply to any required class excursion, with the exception that some class excursions cannot accommodate any tardiness, and students risk being marked as absent if they fail to be present at the appointed time.

Unexcused absences will lead to the following penalties:



<i>Percentage of Total Course Hours Missed</i>	<i>Equivalent Number of Open Campus Semester classes</i>	<i>Minimum Penalty</i>
Up to 10%	1 content classes, or up to 2 language classes	Participation graded as per class requirements
10 – 20%	2 content classes, or 3-4 language classes	Participation graded as per class requirements; written warning
More than 20%	3 content classes, or 5 language classes	Automatic course failure , and possible expulsion

Weekly Schedule

NOTE: this schedule is subject to change at the discretion of the instructor to take advantage of current experiential learning opportunities.

Week 1 Review of Basics and Integration

Session 1.1 Calculus 1 Review - Basics

Students review basics of Pre-Calculus and Calculus 1 with the instructor. Groups of students will solve review problems together and the entire class will review historic and contemporary applications for material covered in Calculus 1. Students will visit local businesses and walk the city with their instructor to explore how Calculus impacts the daily life of local people.

Readings: Appendix C: Review of Pre-Calculus and assigned problems

Watch: Heys, J. 2012. What is Calculus Used For?
https://www.youtube.com/watch?v=_ldra8rVS1I

Week 2 Applications and Techniques of Integration

Quiz 1

Session 2.1 Integration

Students will approximate areas, using sigma notation to calculate sums and powers of integers, the sum of rectangular areas to approximate area under a curve and use Riemann sums to approximate area. They will define the definite integral, explain integrands, limits of integration and variable of integration, explain when a function is integrable, describe the relationship between definite integral and net area, use geometry and properties of definite integrals to evaluate them and calculate the average value of a function. They define the Mean Value Theorem for Integrals, the Fundamental Theorem and evaluate derivatives of integrals. Students will also explain the relationship between differentiation and integration, apply the basic integration formulas, the net



change theorem and apply the integrals of odd and even functions. They will use substitution to evaluate indefinite and definite integrals and integrate functions with exponents, logarithmic and trigonometric functions.

Readings: Chapter 1 Integration, with assigned problems

Due: Calculus 1 Review Problem Set and Excursion 1 essay

Session 2.2 Applications of Integration

Students determine the area of a region between two curves using integration, find the area of a compound region and determine the area of a region between two curves. They determine the volume of a solid by integrating cross sections using slicing, disk and washer methods. They calculate the volume of a solid of revolution using cylindrical shells and explore other methods for doing the same. Students also determine the length of a curve between two points and the surface area of a solid of revolution. They determine the mass of one and two-dimensional objects, calculate work done by a variable force acting along a line, by pumping a liquid from one height to another and the hydrostatic force against a submerged vertical plate. Students also find the center of mass of objects along a line, of a thin plate and apply the theorem of Pappus for volume. They define natural logarithm as an integral, find its derivative and express general logarithmic and exponential functions as natural logarithms and exponentials. They apply this to exponential growth and decay. Finally, they apply derivative and integral formulas to hyperbolic functions.

Reading: Chapter 2 Applications of Integration and assigned problems

Due: Problem Set 2

Week 3 Differential Equations

Quiz 2

Session 3.1 Techniques of Integration.

Students explain when and how to use integration by parts. They solve integration problems involving products and powers of sine, cosine and tangent, using reduction formulas to solve trigonometric integrals. They will also solve integration problems involving the square root of a sum or differences of two squares. They will integrate a rational function using partial fractions, recognize simple, repeated and quadratic factors in a rational function, use a table of integrals to solve integration problems and use a computer algebra system (CAS) to solve integration problems. Students will approximate the value of a definite integral using midpoint and trapezoidal rules, determine the absolute and relative error using numerical integration and an error-bound formula, recognize with the midpoint and trapezoidal rules over or underestimate the true value and use Simpson's rule to approximate the value of a definite integral to a given accuracy.



They will also evaluate an integral over an infinite interval and use the comparison theorem to determine whether a definite integral is convergent.

Readings: Chapter 3 Techniques of Integration and assigned problems.

Due: Problem Set 3

Session 3.2 Introduction to Differential Equations - Basics

Students recognize the order of a differential equation, explain what is meant by a solution to a differential equation, distinguish between the general solution and a particular solution to a differential equation, identify an initial-value problem and identify whether a given function is a solution to a differential equation or an initial-value problem. They draw the direction field for a given first-order equation as well as a solution curve and use Euler's Method to approximate the solution to a first-order differential equation.

Readings: Chapter 4 Introduction to Differential Equations, pp. 351-380 and assigned problems.

Due: Problem Set 4

Session 3.3 Differential Equations – Separable Equations, the Logistic Equation and First Order Linear Equations

Students use separation of variables to solve a differential equation and solve applications using separation variables. They describe the concept of environmental carrying capacity in the logistic model of population growth, draw a directional field for a logistic equation and interpret the solution curves and solve a logistic equation and interpret the results in light of human population and global carrying capacity. They will write a first-order linear differential equation in standard form, find an integrating factor and use it to solve a first-order linear differential equation and solve applied problems involving first-order linear differential equations.

Readings: Chapter 4 Introduction to Differential Equations, pp. 381-425 and assigned problems and Dobyns, H.F., 2017. Estimating aboriginal American population: An appraisal of techniques with a new hemispheric estimate. In *The Atlantic Slave Trade* (pp. 163-186). Routledge.

Due: Problem Set 5

Week 4 Sequences and Series

Quiz 3

Session 4.1 Sequences and Series



Students will find the formula for the general term of a sequence, calculate the limit of a sequence if it exists and determine the convergence or divergence of a given sequence. They will explain the meaning of the sum of an infinite series, calculate the sum of a geometric series and evaluate a telescoping series. Students will use the divergence test to determine if a series converges or diverges, use the integral test to determine convergence of a series and estimate the value of a series by finding bounds on its remainder term.

Readings: Chapter 5 Sequences and Series, pp. 427-484 and assigned problems.

Due: Problem Set 6

Session 4.2 Visit to Mathematics Department of local university.

Here, students will speak with a representative mathematician about current projects connecting math to society. Students will have a tour, speak with several mathematicians and discuss how math, science, society and culture interact. They will also use the ratio and root tests to determine absolute convergence of a series and describe a strategy for testing the convergence of a given series.

Reading: Jurdak, M.E., 2014. Religion and Language as Cultural Carriers and Barriers in Mathematics Education--Revisited. *Journal of Humanistic Mathematics*, 4(2), pp.47-57.

Due: Problem Set 7

Session 4.3 Comparison Tests, Alternating Series, Ratio and Root Tests.

Students use the comparison test to test a series for convergence and use the limit comparison test to determine convergence of a series. They use the alternating series test to test an alternating series for convergence, estimate the sum of an alternating series and explain the meaning of absolute convergence and conditional convergence.

Reading: Chapter 5 Sequences and Series, pp. 485-530 and assigned problems.

Due: Problem Set 8, Essay 2: Math, Science and Society

Week 5 Power Series

Quiz 4

Session 5.1 Power Series

Students identify a power series and provide examples, determine the radius of



convergence and interval of convergence of a power series and use a power series to represent a function. They then combine power series by addition and subtraction, create a new power series by multiplication by a power of the variable or a constant, or by substitution. They multiply two power series together and differentiate and integrate power series term-by-term.

Readings: Chapters 6 Power Series, pp. 531-560 and assigned problems.

Due: Problem Set 9

Session 5.2 Taylor and Maclaurin Series

Students will describe the procedure for finding a Taylor polynomial of a given order of a function, explain the meaning and significance of a Taylor's theorem with remainder and estimate the remainder for a Taylor series approximation of a given function. They write terms of the binomial series, recognize the Taylor series expansions of common functions, recognize and apply techniques to find the Taylor series for a function and use Taylor series to solve differential equations as well as nonelementary integrals.

Readings: Chapters 5 Integration, pp. 561-600 and assigned problems

Due: Problem Set 10

Session 5.3 Problem Solving Workshop

Students will be given real world scenarios by the instructor. Students will work in groups to solve these more complex and nuanced problems. They will discuss the relevance of their findings and explore other applications.

Due: Problem Set 11

Week 6 Parametric Equations and Polar Coordinates

Quiz 5

Session 6.1 Parametric Equations

Students will plot a curve described by parametric equations, convert the parametric equations of a curve into the form $y = f(x)$, recognize the parametric equations of basic curves, such as a line a circle and cycloid. They determine derivatives and equations of tangents for parametric curves, find the area under a parametric curve, use the equation for arc length of a parametric curve, apply the formula for surface area to a volume generated by a parametric curve.

Readings: Chapter 7 Application of Integration, pp. 605-640 and assigned problems.



Due: Problem Set 12

Session 6.2 Polar Coordinates

Students will locate points in a plane using polar coordinates, convert points between rectangular and polar coordinates, sketch polar curves from given equations, convert equations between rectangular and polar coordinates, and identify symmetry in polar curves and equations. Students apply the formula for area of a region in polar coordinates and determine the arc length of a polar curve. They identify the equation of a parabola in standard form with given focus and directrix, the equation of an ellipse and hyperbola, recognize a parabola, ellipse or hyperbola from its eccentricity value, write the polar equation of a conic section with eccentricity e and identify when a general equation of degree two is a parabola, ellipse or hyperbola.

Reading: Chapter 7 Application of Integration, pp. 642-690 and assigned problems.

Session 6.3 Problem Workshop, Applications of Calculus

Students will review and work on problems with their instructor. Further, students will research and present recent applications of calculus to one another. There will be a discussion of new applications and future applications of Calculus and how its implementation can further benefit society.

Watch: Charles, R. 2014. The Next Frontier in Mathematics
https://www.youtube.com/watch?v=cFg4Fmw_kOE

Due: Final Problem Set, Final Quiz

Course Materials

Course Textbook

Herman, E. and G. Strang. 2017. Calculus Volume 2. Houston, TX: Openstax CNX

Readings

Dobyns, H.F., 2017. Estimating aboriginal American population: An appraisal of techniques with a new hemispheric estimate. In *The Atlantic Slave Trade* (pp. 163-186). Routledge

Jurdak, M.E., 2014. Religion and Language as Cultural Carriers and Barriers in Mathematics Education--Revisited. *Journal of Humanistic Mathematics*, 4(2), pp.47-57

Online Resources



Uses of Calculus in Everyday Life <https://sciencing.com/uses-calculus-real-life-8524020.html>